Parallelism 2

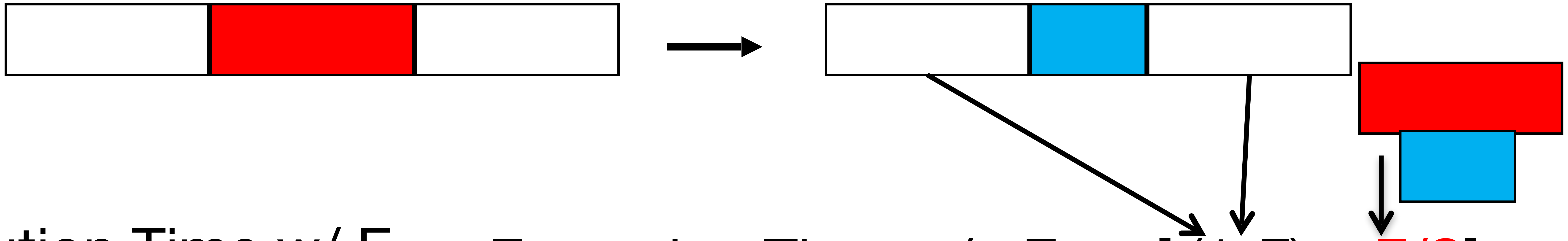
Berkeley EECS
ELECTRICAL ENGINEERING & COMPUTER SCIENCES

Big Idea Reminder: Amdahl's (Heartbreaking) Law

- Speedup due to enhancement E is

$$\text{Speedup w/ E} = \frac{\text{Exec time w/o E}}{\text{Exec time w/ E}}$$

- Suppose that enhancement E accelerates a fraction F (F <1) of the task by a factor S (S>1) and the remainder of the task is unaffected



Execution Time w/ E = Execution Time w/o E × [(1-F) + F/S]

Speedup w/ E = 1 / [(1-F) + F/S]

Big Idea: Amdahl's Law

$$\text{Speedup} = \frac{1}{(1 - F) + \frac{F}{S}}$$

Non-speed-up part

Speed-up part

Example: the execution time of half of the program can be accelerated by a factor of 2.
What is the program speed-up overall?

$$\frac{1}{0.5 + \frac{0.5}{2}} = \frac{1}{0.5 + 0.25} = 1.33$$

Example #1: Amdahl's Law

Speedup w/ E = 1 / [(1-F) + F/S]

- Consider an enhancement which runs 20 times faster but which is only usable 25% of the time

  Speedup w/ E = 1/(.75 + .25/20) = 1.31

- What if its usable only 15% of the time?

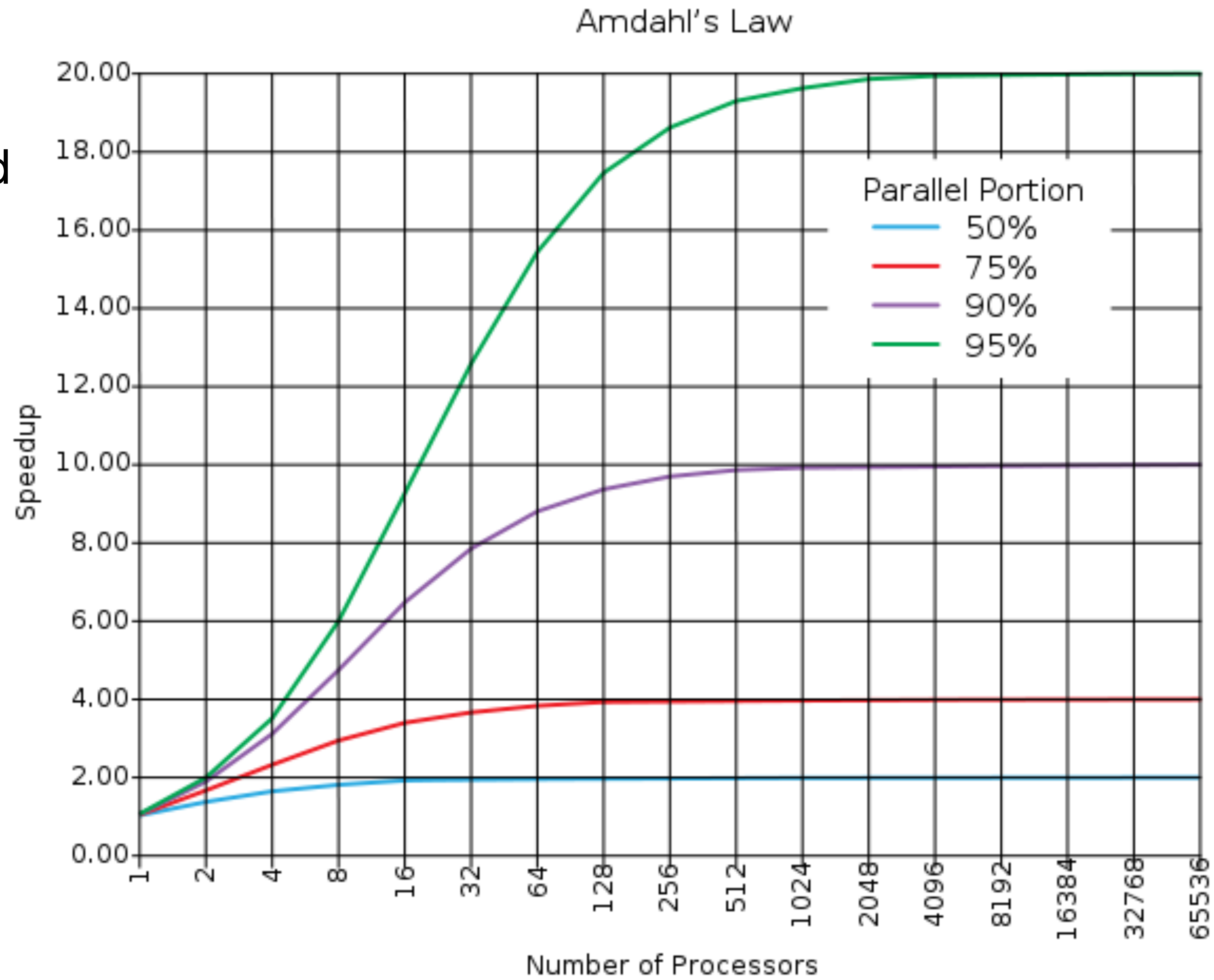
  Speedup w/ E = 1/(.85 + .15/20) = 1.17

- Amdahl's Law tells us that to achieve linear speedup with 100 processors, none of the original computation can be scalar!
- To get a speedup of 90 from 100 processors, the percentage of the original program that could be scalar would have to be 0.1% or less

  Speedup w/ E = 1/(.001 + .999/100) = 90.99

Amdahl’s Law

If the portion of the program that can be parallelized is small, then the speedup is limited



The non-parallel portion limits the performance

Strong and Weak Scaling

- To get good speedup on a parallel processor while keeping the problem size fixed is harder than getting good speedup by increasing the size of the problem.
  - ***Strong scaling***: when speedup can be achieved on a parallel processor without increasing the size of the problem
  - ***Weak scaling***: when speedup is achieved on a parallel processor by increasing the size of the problem proportionally to the increase in the number of processors
- ***Load balancing*** is another important factor: every processor doing same amount of work
  - Just one unit with twice the load of others cuts speedup almost in half

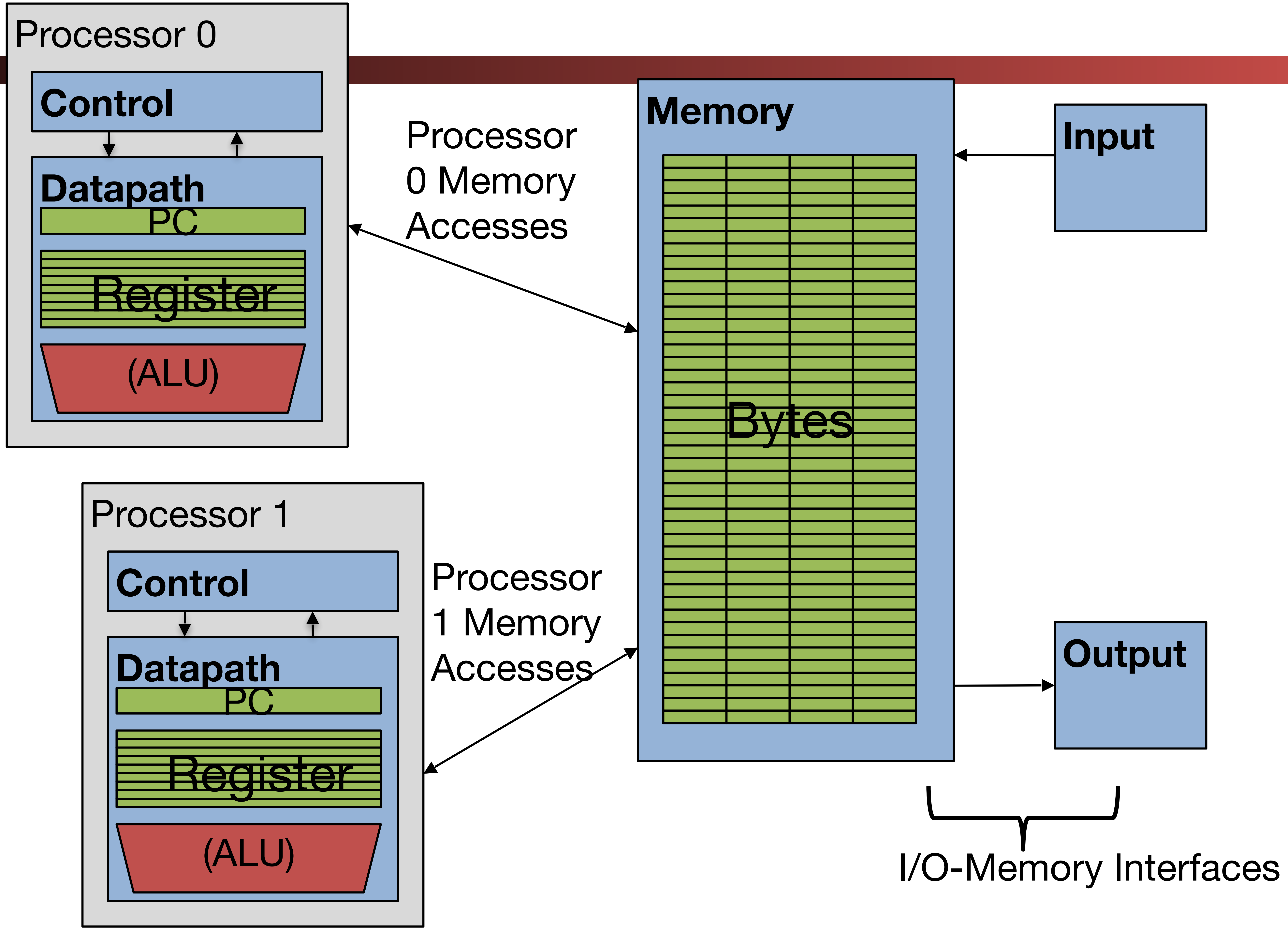
Amdahl's Law In The Real World…

- Lets look at the federal budget:
  - Price of a ***single*** F35: >$100M
    - Air Force ***alone*** wants to buy 33 next year
  - Line item: "Purchase F35 fighter jets for the Air Force:" ~$4.5B
    - This doesn't include the Navy's Air Force's purchases...
      Or the Navy's Army's Air Force's purchases...
  - Line item: "Fund Corporation for Public Broadcasting:" ~$500M
- If you want to reduce the cost of the federal government…
  - Which line item is more significant?

Amdahl's Law and Premature Optimization...

- The runtime of a new program is really...
  - The runtime of the program on all the inputs you ever run it on
  - The time it takes you to ***write*** the program in the first place!
- So don't ***prematurely optimize***
  - Worry about getting things right first, you may never have to optimize it at all
- Likewise, worry about making your code readable and well documented:
  - Since the runtime of a modified version of the program is the runtime on all inputs plus the time it takes you to relearn what you did in order to modify it!

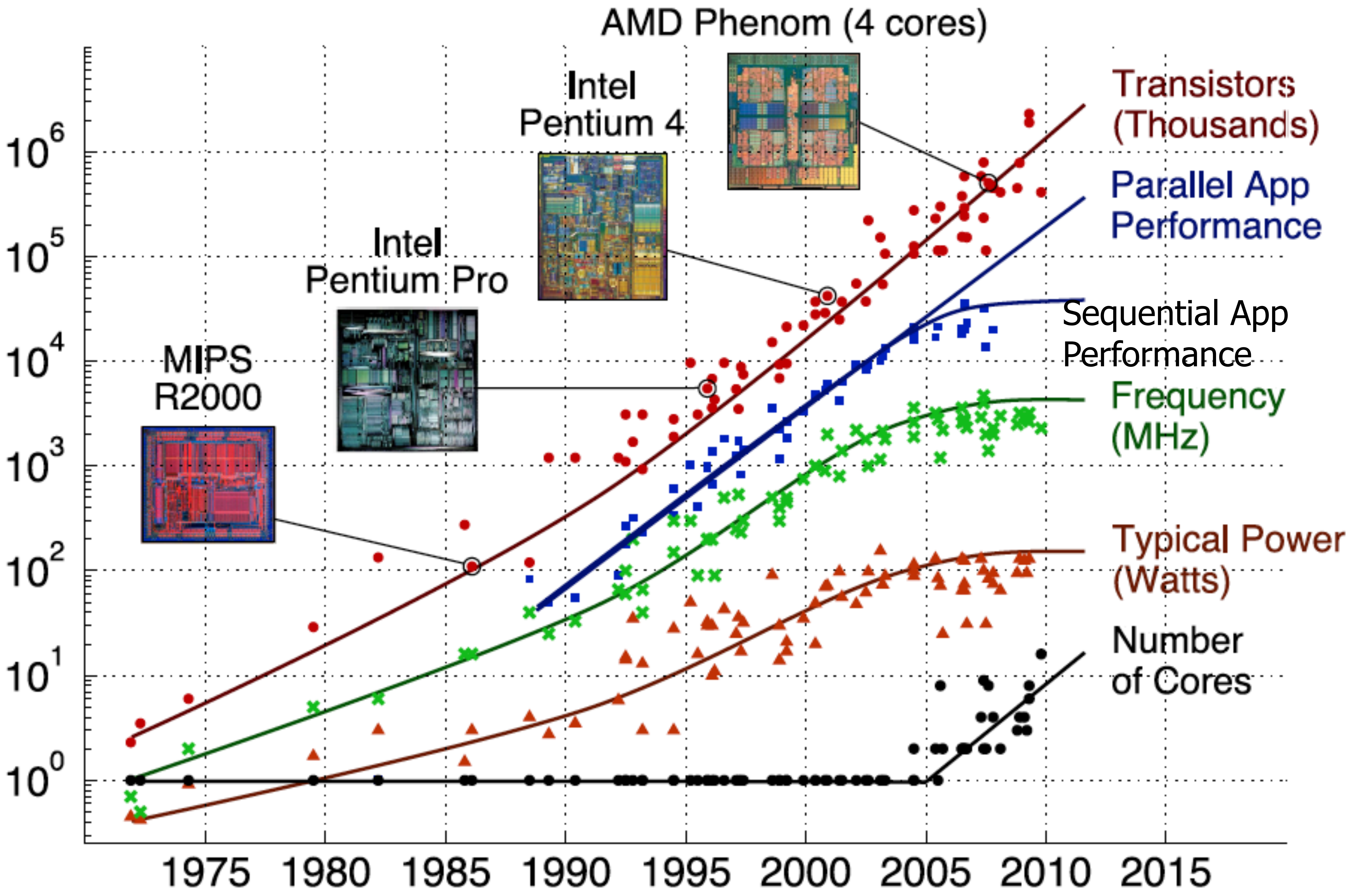
Simple Multiprocessor



Multiprocessor Execution Model

- Each processor has its own PC and executes an independent stream of instructions (MIMD)
- Different processors can access the same memory space
  - Processors can communicate via shared memory by storing/loading to/from common locations
- Two ways to use a multiprocessor:
  - Deliver high throughput for independent jobs via job-level parallelism
    - E.g. your operating system & different programs
  - Improve the run time of a single program that has been specially crafted to run on a multiprocessor - a parallel-processing program
- Use term core for processor ("Multicore") because "Multiprocessor Microprocessor" too redundant

Transition to Multicore



Data partially collected by M. Horowitz, F. Labonte, O. Shacham, K. Olukotun, L. Hammond

Parallelism the Only Path to Higher Performance

- Sequential processor performance not expected to increase much:
  - We pretty much hit a brick wall a few years back in our ability to improve single-thread performance: Apple got a one-time boost with transition to ARM
- If want apps with more capability we have to embrace parallel processing (SIMD and MIMD)
- In mobile systems, use multiple cores and GPUs
  - All iPhones starting with the 4s are multicore
  - iPhone 12 CPU is 6 cores!
    - Two cores very fast: Burn lots of power but very good sequential performance
    - Four cores power efficient: Lower sequential performance but better ops/joule
    - Plus a 4 core GPU
    - Plus a 16 core processor for machine learning (optimized for 16b floating point!)
- In warehouse-scale computers, use multiple nodes, and all the MIMD/SIMD capability of each node

Comparing Types of Parallelism…

- SIMD-type parallelism (Data Parallel)
  - A SIMD-favorable problem can map easily to a MIMD-type fabric
  - SIMD-type fabrics generally offer a much higher ***throughput per $***
    - Much simpler control logic
    - Classic example: Graphics cards are massive supercomputers compared to the CPU: ***teraflops*** rather than gigaflops: so 500x-1000x performance!
  - Common approach is "vector" like we see with Intel AVX:
    - EG, 512b vector of double-precision floating point: 8 elements at a time
- MIMD-type parallelism (data-dependent Branches!)
  - A MIMD-favorable problem ***will not map easily*** to a SIMD-type fabric

Multiprocessors and You

- Only path to performance is parallelism
  - Clock rates flat or declining
  - CPI generally flat
  - SIMD now ~4-16 words wide on the CPU
  - SIMD accelerators even more
    - Nvidia GP100 GPU: 5 TFLOPs of 64b Floating Point, 10 for 32b FP
      1792 CUDA cores for 64b Floating Point (3584 for 32b)
  - MIMD: Add 2 cores every 2 years: 2, 4, 6, 8, 10, …
- Key challenge is to craft parallel programs that have high performance on multiprocessors as the number of processors increase – i.e., that scale
  - Scheduling, load balancing, time for synchronization, overhead for communication
- If you can scale up you can ***then scale down***

Threads

- ***Thread***: a sequential flow of instructions that performs some task
- Each thread has a PC + processor registers and accesses the ***shared memory of the process***
- Each core provides one or more ***hardware threads*** that actively execute instructions
  - Common Intel chips support 2 threads/core
    - So a 4 core Intel processor can support 8 hardware threads
  - The RPi4 has only 1 thread per core -> 4 cores -> 4 hardware threads
- Operating system multiplexes multiple ***software threads*** onto the available ***hardware threads***

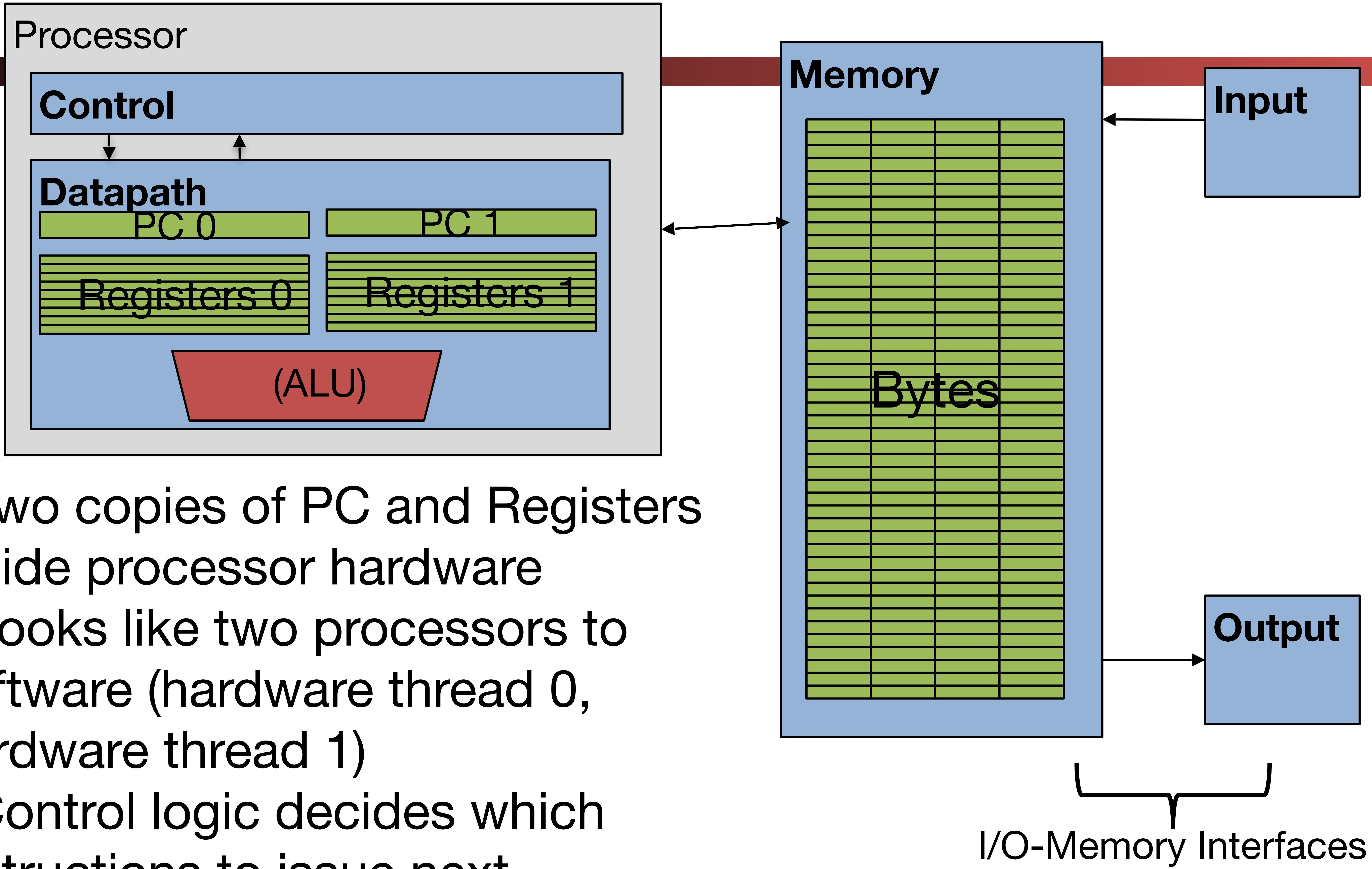
Operating System Threads

- Give the illusion of many active threads by time-multiplexing software threads onto hardware threads
- Remove a software thread from a hardware thread by interrupting its execution and saving its registers and PC into memory
  - Also if one thread is blocked waiting for network access or user input can switch to another thread
- Can make a different software thread active by loading its registers into a hardware thread's registers and jumping to its saved PC

Hardware Multithreading

- Basic idea: Processor resources are expensive and should not be left idle
  - Long memory latency to memory on cache miss is the biggest one
- Hardware switches threads to bring in other useful work while waiting for cache miss
  - Cost of thread context switch must be much less than cache miss latency
- Put in redundant hardware so don't have to save context on every thread switch:
  - PC, Registers
- Attractive for apps with abundant TLP
  - Commercial multi-user workloads
- Intel calls this HyperThreading
  - Will actually issue from two threads at the same time!

Hardware Multithreading



- Two copies of PC and Registers inside processor hardware
- Looks like two processors to software (hardware thread 0, hardware thread 1)
- Control logic decides which instructions to issue next
  - Can even mix from dif threads

Multithreading vs. Multicore

- Multithreading => Better Utilization
  - ≈1% more hardware, 1.10X better performance?
  - Share integer adders, floating-point units, all caches (***L1 I$, L1 D$, L2$, L3$***), Memory Controller
- Multicore => Duplicate Processors
  - ≈50% more hardware, ≈2X better performance?
  - Share ***outer caches*** (L2$ or just L3$), Memory Controller
- Modern Intel machines do both
  - Multiple cores with 2 threads per core
- ARM machines don't bother
  - Performance win doesn't appear to be worth the complexity

Nick's MacBook Pro
MacBookPro 13" (2020)

- `/usr/sbin/sysctl -a | grep hw\.`

...

hw.physicalcpu: 4

hw.logicalcpu: 8

...

hw.cpufrequency = 2,000,000,000

hw.memsize = 34,359,738,368

hw.cachelinesize = 64

hw.l1icachesize: 32,768

hw.l1dcachesize: 49,152

hw.l2cachesize: 524,288

hw.l3cachesize: 6,291,456

Nick's Zoom-Cave Beast

- AMD Ryzen 9 3900X 12 core CPU
  - 2 threads/core
- Nvidia 2080 GPU
  - 2944 CUDA SIMD processor cores
- Gratuitous BlinkenLights...
  - Hey, those are the factory lights on the CPU and GPU...
  - But I did get a transparent case...

Nick's $45 Raspberry Pi 4…

- Quad-Core processor
  - 1 thread/core
  - 3-issue out-of-order superscalar, 16 stage pipeline
  - 128b wide SIMD/vector instructions (4x single precision floating point)
- 512 KB shared L2 cache
  - L1 I$ is 48 KB
    L1 D$ is 32 KB
- 4 GB RAM
- Gb Ethernet, 802.11, Bluetooth

- Even the smallest and cheapest systems are now heavily parallel
  - OK full kit cost $75… With HDMI cable, power supply, case, SD-card

Lastest modern processors: Big/Little design

- You need "big" processors for both single threaded and multi-threaded performance
  - After all, you don't want to wait around...
- But such processors are ***very*** inefficient
  - Lots of power, lots of silicon
  - And a lot of time you don't need a big processor, because you don't need the performance
- Modern big/little design
  - Intel Alder Lake: i9 version: 8 performance cores (with 2 threads/core) + 4 efficiency cores
    - Efficiency cores only support one thread/core, and are designed in a block of 4 with a shared L2 cache
  - Apple M1 Pro: 8 performance cores, 2 efficiency cores
    - All cores are single thread/core

OpenMP

- OpenMP is a language extension used for multi-threaded, shared-memory parallelism
  - Compiler Directives (inserted into source code)
  - Runtime Library Routines (called from your code)
  - Environment Variables (set in your shell)
- Portable
- Standardized
  - But beyond the C language itself
- Easy to compile: cc –fopenmp name.c

Shared Memory Model with Explicit Thread-based Parallelism

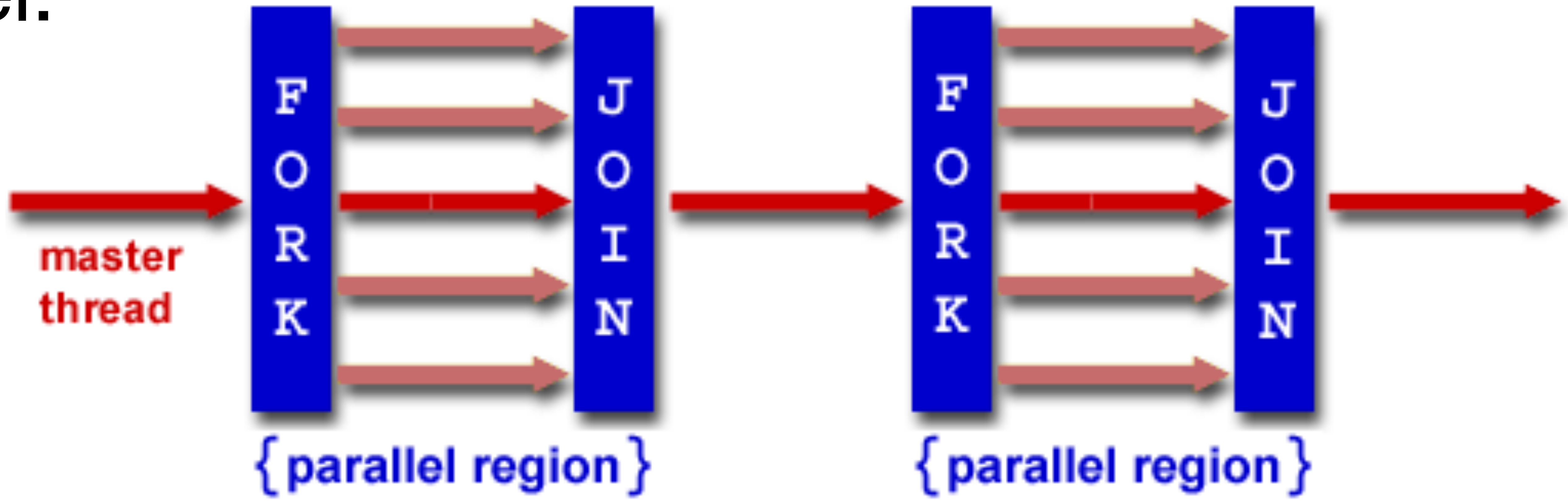
- Multiple threads in a shared memory environment, explicit programming model with full programmer control over parallelization
- ***Pros***:
  - Takes advantage of shared memory, programmer need not worry (that much) about data placement
  - Compiler directives are simple and easy to use
  - Legacy serial code does not need to be rewritten
- ***Cons***:
  - Code can only be run in shared memory environments
  - Compiler must support OpenMP (e.g. gcc 4.2)
  - Amdahl's law is gonna get you after not too many cores...

OpenMP in CS61C

- OpenMP is built on top of C, so you don't have to learn a whole new programming language
  - Make sure to add `#include <omp.h>`
  - Compile with flag: `gcc -fopenmp`
  - Mostly just a few lines of code to learn
- You will NOT become experts at OpenMP
  - Use slides as reference, will learn to use in lab
- Key ideas:
  - Shared vs. Private variables
  - OpenMP directives for parallelization, work sharing, synchronization

OpenMP Programming Model

- Fork - Join Model:



- OpenMP programs begin as single process (master thread) and executes sequentially until the first parallel region construct is encountered
  - `FORK`: Master thread then creates a team of parallel threads
    - Statements in program that are enclosed by the parallel region construct are executed in parallel among the various threads
  - `JOIN`: When the team threads complete the statements in the parallel region construct, they synchronize and terminate, leaving only the master thread

OpenMP Extends C with Pragmas

- ***Pragmas*** are a preprocessor mechanism C provides for language extensions
- Commonly implemented pragmas:
  structure packing, symbol aliasing, floating point exception modes (not covered in 61C)
- Good mechanism for OpenMP because compilers that don't recognize a pragma are supposed to ignore them
  - Runs on sequential computer even with embedded pragmas

`parallel` Pragma and Scope

- Basic OpenMP construct for parallelization:

```
#pragma omp parallel
  {
    /* code goes here */
  }
```

This is annoying, but curly brace ***MUST*** go on separate line from #pragma

  - Each thread runs a copy of code within the block
  - Thread scheduling is non-deterministic
- OpenMP default is shared variables
  - To make private, need to declare with pragma:
- `#pragma omp parallel private (x)`

Thread Creation

- How many threads will OpenMP create?
- Defined by **`OMP_NUM_THREADS`** environment variable (or code procedure call)
  - Set this variable to the maximum number of threads you want OpenMP to use
  - Usually equals the number of physical cores * number of threads/core in the underlying hardware on which the program is run
    - EG, RPi 4 has 4 threads by default

What Kind of Threads?

- OpenMP threads are operating system (software) threads.
- OS will multiplex requested OpenMP threads onto available hardware threads.
- Hopefully each gets a real hardware thread to run on, so no OS-level time-multiplexing.
- But other tasks on machine can also use hardware threads!
  - And you may want more threads than hardware if you have a lot of I/O so that while waiting for I/O other threads can run
- Be careful when timing results!

OMP_NUM_THREADS

- OpenMP intrinsic to set number of threads:
  `omp_set_num_threads(x);`
- OpenMP intrinsic to get number of threads:
  `num_th = omp_get_num_threads();`
- OpenMP intrinsic to get Thread ID number:
  `th_ID = omp_get_thread_num();`

Parallel Hello World


```
#include <stdio.h>
#include <omp.h>
int main () {
  int nthreads, tid;
  /* Fork team of threads with private var tid */
  #pragma omp parallel private(tid)
  {
    tid = omp_get_thread_num(); /* get thread id */
    printf("Hello World from thread = %d\n", tid);
    /* Only master thread does this */
    if (tid == 0) {
      nthreads = omp_get_num_threads();
      printf("Number of threads = %d\n", nthreads);
    }
  }  /* All threads join master and terminate */
}
```

Data Races and Synchronization

- Two memory accesses form a ***data race*** if different threads attempts to access the same location, and at least one is a write, and they occur one after another
- If there is a data race, result of program can vary depending on chance (which thread first?)
- Avoid data races by synchronizing writing and reading to get deterministic behavior
- Synchronization done by user-level routines that rely on hardware synchronization instructions
  - (more later)

Analogy: Buying ~~Beer~~ Milk In the After Times...

- Your fridge has no milk. You and your roommate will return from classes at some point and check the fridge
- Whoever gets home first will check the fridge, go and buy milk, and return
- What if the other person gets back while the first person is buying milk?
  - You've just bought twice as much milk as you need!
- It would've helped to have left a note…

Lock Synchronization (1/2)

- Use a "Lock" to grant access to a region (***critical section***) so that only one thread can operate at a time
  - Need all processors to be able to access the lock, so use a location in shared memory as the lock
- Processors read lock and either wait (if locked) or set lock and go into critical section
  - 0 means lock is free / open / unlocked / lock off
  - 1 means lock is set / closed / locked / lock on

Lock Synchronization (2/2)

- Pseudocode:

```
Check lock
Set the lock
Critical section
(e.g. change shared variables)
Unset the lock
```

Can loop/idle here if locked

Possible Lock Implementation

- Lock (a.k.a. busy wait)

```
Get_lock:                    # s0 -> addr of lock
       addi t1,x0,1          # t1 = Locked value
Loop:  lw t0,0(s0)           # load lock
       bne t0,x0,Loop        # loop if locked
Lock:  sw t1,0(s0)           # Unlocked, so lock
```

- Unlock

```
Unlock:
      sw x0,0(s0)
```

- Any problems with this?

Possible Lock Problem

- Thread 1

```
      addi t1,x0,1
Loop: lw t0,0(s0)

      bne t0,x0,Loop

Lock: sw t1,0(s0)
```

- Thread 2

```
      addi t1,x0,1
Loop: lw t0,0(s0)

      bne t0,x0,Loop

Lock: sw t1,0(s0)
```

Time

Both threads think they have set the lock!
*Exclusive access **not guaranteed**!*

Hardware Synchronization

- Hardware support required to prevent an interloper (another thread) from changing the value
  - Atomic read/write memory operation
    - No other access to the location allowed between the read and write
- How best to implement in software?
  - Single instr? Atomic swap of register ↔ memory
  - Pair of instr? One for read, one for write
- Needed even on uniprocessor systems
  - After all, Interrupts happen, and can trigger thread context switches...

Synchronization in RISC-V option one: Read/Write Pairs

- Load reserved: `lr rd, rs`
  - Load the word pointed to by `rs` into `rd`, and add a reservation
- Store conditional: `sc rd, rs1, rs2`
  - Store the value in `rs2` into the memory location pointed to by `rs1`, only if the reservation is still valid and set the status in `rd`
    - Returns 0 (success) if location has not changed since the `lr`
    - Returns nonzero (failure) if location has changed:
      Actual store will not take place

Synchronization in RISC-V Example

- Atomic swap (to test/set lock variable)

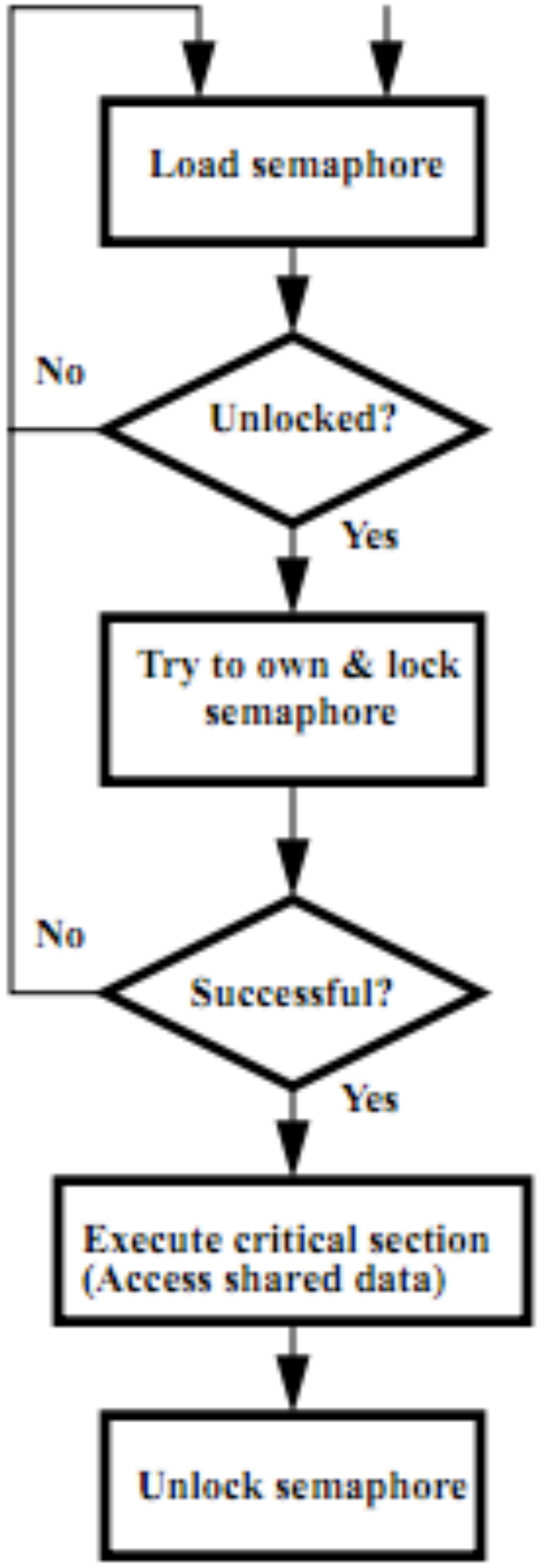
  Exchange contents of register and memory:
  s4 ↔ Mem(s1)

```
try:
      lr   t1, s1          #load reserved
      sc   t0, s1, s4      #store conditional
      bne  t0,x0,try       #loop if sc fails
      add  s4,x0,t1        #load value in s4
```

`sc` would fail if another threads executes `sc` here

Test-and-Set

- In a single atomic operation:
  - ***Test*** to see if a memory location is set (contains a 1)
  - ***Set*** it (to 1) if it isn't (it contained a zero when tested)
    - Otherwise indicate that the Set failed, so the program can try again
  - While accessing, no other instruction can modify the memory location, including other Test-and-Set instructions
- Useful for implementing lock operations



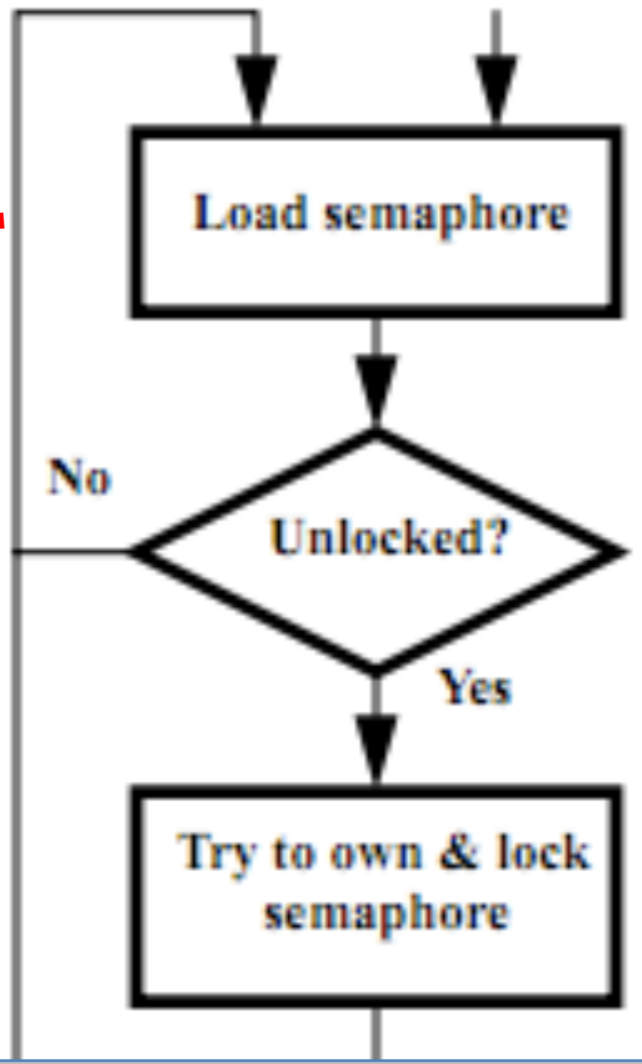
Test-and-Set in RISC-V using lr/sc

- Example: RISC-V sequence for implementing a T&S at (s1)

```
      li  t2, 1
Try:  lr  t1, s1
      bne t1, x0, Try
      sc  t0, s1, t2
      bne t0, x0, Try
Locked:

      # critical sec

Unlock:
      sw x0,0(s1)
```



Idea is that not for programmers to use this directly, but as a tool for enabling implementation of parallel libraries

RISC-V Alternative: Atomic Memory Operations

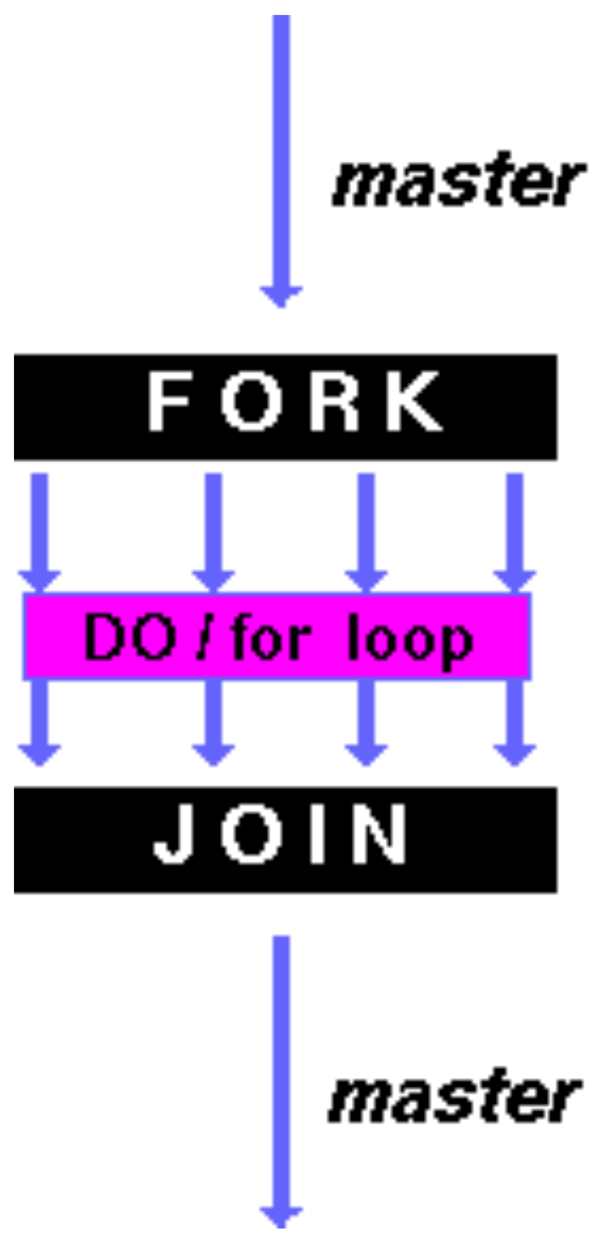
- Three instruction rtype instructions
  - Swap, and, add, or, xor, max, min
    `AMOSWAP rd,rs2,(rs1)`
    `AMOADD rd,rs2,(rs1)`
- Take the value ***pointed to*** by rs1
  - Load it into **`rd`**
  - Apply the operation to that value with the contents in **`rs2`**
    - if **`rs2`** == **`rd`**, use the *old* value in **`rd`**
  - store the result back to where **`rs1`** is pointed to
- This allow atomic swap as a primitive
  - It also allows "reduction operations" that are common to be efficiently implemented

OpenMP Building Block: `for` loop rather than just the parallel block

- `for (i=0; i<max; i++) zero[i] = 0;`
- Breaks for loop into chunks, and allocate each to a separate thread
  - e.g. if max = 100 with 2 threads:
    assign 0-49 to thread 0, and 50-99 to thread 1
- Must have relatively simple “shape” for an OpenMP-aware compiler to be able to parallelize it
  - Necessary for the run-time system to be able to determine how many of the loop iterations to assign to each thread:
    Not a good idea to be changing the loop bounds in the loop itself
- No premature exits from the loop allowed
  - i.e. No break, return, exit, goto statements, changing loop bounds, instead just simple `for` and `while` loops

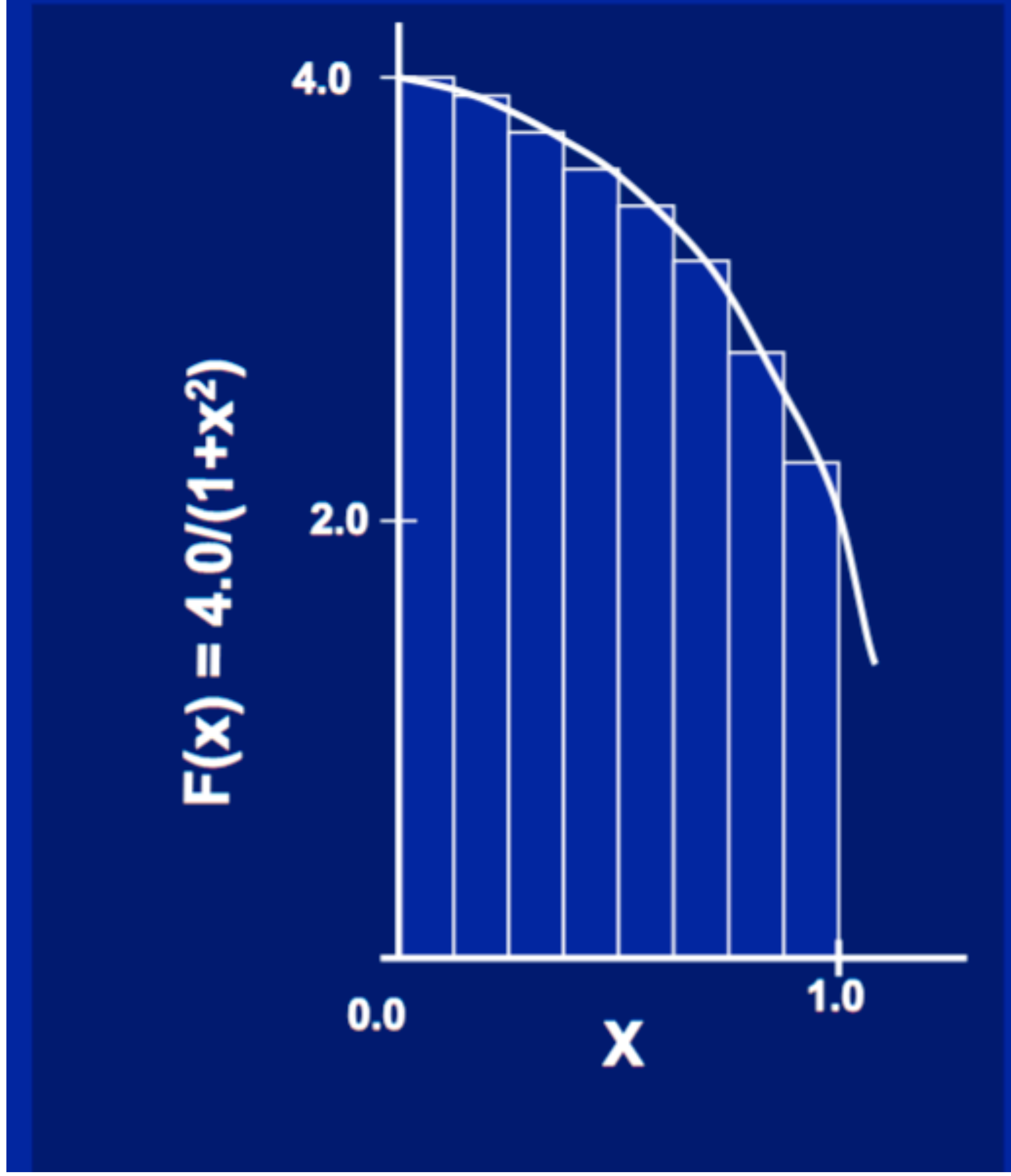
OpenMP `parallel for` pragma

- `#pragma omp parallel for`
  `for (i=0; i<max; i++) zero[i] = 0;`
- Master thread creates additional threads, each with a separate execution context
- All variables declared outside for loop are shared by default, except for loop index which is ***implicitly*** private per thread
- Implicit “barrier” synchronization at end of for loop
- Divide index regions sequentially per thread
  - Thread 0 gets 0, 1, …, (max/n)-1;
  - Thread 1 gets max/n, max/n+1, …, 2*(max/n)-1



Example 2: Computing π

Numerical Integration



Mathematically, we know that:

$$\int_0^1 \frac{4.0}{(1+x^2)}\,dx = \pi$$

We can approximate the integral as a sum of rectangles:

$$\sum_{i=0}^{N} F(x_i)\Delta x \approx \pi$$

Where each rectangle has width Δx and height $F(x_i)$ at the middle of interval i.

http://openmp.org/mp-documents/omp-hands-on-SC08.pdf

Working Parallel π without a for loop


```
#include <stdio.h>
#include <omp.h>

void main () {
    const int NUM_THREADS = 4;
    const long num_steps = 10;
    double step = 1.0/((double)num_steps);
    double sum[NUM_THREADS];
    for (int i=0; i<NUM_THREADS; i++) sum[i] = 0;
    omp_set_num_threads(NUM_THREADS);
#pragma omp parallel
    {
        int id = omp_get_thread_num();
        for (int i=id; i<num_steps; i+=NUM_THREADS) {
            double x = (i+0.5) *step;
            sum[id] += 4.0*step/(1.0+x*x);
            printf("i =%3d,  id =%3d\n", i, id);
        }
    }
    double pi = 0;
    for (int i=0; i<NUM_THREADS; i++) pi += sum[i];
    printf ("pi = %6.12f\n", pi);
}
```

Trial Run

```
#include <stdio.h>
#include <omp.h>

void main () {
    const int NUM_THREADS = 4;
    const long num_steps = 10;
    double step = 1.0/((double)num_steps);
    double sum[NUM_THREADS];
    for (int i=0; i<NUM_THREADS; i++) sum[i] = 0;
    omp_set_num_threads(NUM_THREADS);
#pragma omp parallel
    {
        int id = omp_get_thread_num();
        for (int i=id; i<num_steps; i+=NUM_THREADS) {
            double x = (i+0.5) *step;
            sum[id] += 4.0*step/(1.0+x*x);
            printf("i =%3d,  id =%3d\n", i, id);
        }
    }
    double pi = 0;
    for (int i=0; i<NUM_THREADS; i++) pi += sum[i];
    printf ("pi = %6.12f\n", pi);
}
```

```
i =   1,   id =   1
i =   0,   id =   0
i =   2,   id =   2
i =   3,   id =   3
i =   5,   id =   1
i =   4,   id =   0
i =   6,   id =   2
i =   7,   id =   3
i =   9,   id =   1
i =   8,   id =   0
pi = 3.142425985001
```

Scale up: `num_steps = 10⁶`


```c
#include <stdio.h>
#include <omp.h>

void main () {
    const int NUM_THREADS = 4;
    const long num_steps = 1000000;
    double step = 1.0/((double)num_steps);
    double sum[NUM_THREADS];
    for (int i=0; i<NUM_THREADS; i++) sum[i] = 0;
    omp_set_num_threads(NUM_THREADS);
#pragma omp parallel
    {
        int id = omp_get_thread_num();
        for (int i=id; i<num_steps; i+=NUM_THREADS) {
            double x = (i+0.5) *step;
            sum[id] += 4.0*step/(1.0+x*x);
            // printf("i =%3d,  id =%3d\n", i, id);
        }
    }
    double pi = 0;
    for (int i=0; i<NUM_THREADS; i++) pi += sum[i];
    printf ("pi = %6.12f\n", pi);
}
```

`pi = 3.141592653590`

Can We Parallelize Computing **sum**?


```
#include <stdio.h>
#include <omp.h>

void main () {
    const int NUM_THREADS = 1000;
    const long num_steps = 100000;
    double step = 1.0/((double)num_steps);
    double sum[NUM_THREADS];
    for (int i=0; i<NUM_THREADS; i++) sum[i] = 0;
    double pi = 0;
    omp_set_num_threads(NUM_THREADS);
#pragma omp parallel
    {
        int id = omp_get_thread_num();
        for (int i=id; i<num_steps; i+=NUM_THREADS) {
            double x = (i+0.5) *step;
            sum[id] += 4.0*step/(1.0+x*x);
        }
        pi += sum[id];
    }
    printf ("pi = %6.12f\n", pi);
}
```

Always looking for ways to beat Amdahl's Law …

Summation inside parallel section

- Insignificant speedup in this example, but …
- **pi = 3.138450662641**
- Wrong! And value changes between runs?!
- What's going on?

What’s Going On?


```c
#include <stdio.h>
#include <omp.h>

void main () {
    const int NUM_THREADS = 1000;
    const long num_steps = 100000;
    double step = 1.0/((double)num_steps);
    double sum[NUM_THREADS];
    for (int i=0; i<NUM_THREADS; i++) sum[i] = 0;
    double pi = 0;
    omp_set_num_threads(NUM_THREADS);
#pragma omp parallel
    {
        int id = omp_get_thread_num();
        for (int i=id; i<num_steps; i+=NUM_THREADS) {
            double x = (i+0.5) *step;
            sum[id] += 4.0*step/(1.0+x*x);
        }
        pi += sum[id];
    }
    printf ("pi = %6.12f\n", pi);
}
```

- Operation is really
  `pi = pi + sum[id]`
- What if >1 threads reads current (same) value of **`pi`**, computes the sum, stores the result back to **`pi`**?
- Each processor reads same intermediate value of **`pi`**!
- Result depends on who gets there when
  - A “race” → result is **not deterministic but if we locked this we'd lose almost all speedup**

OpenMP Reduction


- ```
  double avg, sum=0.0, A[MAX]; int i;
  #pragma omp parallel for private ( sum )
  for (i = 0; i <= MAX ; i++) {sum += A[i];}
  avg = sum/MAX;  // bug, we only get the master thread's sum
  ```
- Problem is that we really want sum over all threads!
- ***Reduction***: specifies that 1 or more variables that are private to each thread are subject of reduction operation at end of parallel region:
  reduction(operation:var) where
  - Operation: operator to perform on the variables (var) at the end of the parallel region
  - Var: One or more variables on which to perform scalar reduction: private than combined
- ```
  double avg, sum=0.0, A[MAX]; int i;
  #pragma omp for reduction(+ : sum)
  for (i = 0; i <= MAX ; i++) {sum += A[i];}
  avg = sum/MAX;
  ```

Calculating π Simple Version


```
#include <omp.h>
#define NUM_THREADS 4
static long num_steps = 100000; double step;

void main () {
  int i;    double  x, pi, sum[NUM_THREADS];
  step = 1.0/(double) num_steps;
  #pragma omp parallel private ( i, x )
  {
    int id = omp_get_thread_num();
    for (i=id, sum[id]=0.0; i<num_steps; i=i+NUM_THREADS)
    {
      x = (i+0.5)*step;
      sum[id] += 4.0/(1.0+x*x);
    }
  }
  for(i=1; i<NUM_THREADS; i++)
    sum[0] += sum[i];  pi = sum[0];
  printf ("pi = %6.12f\n", pi);
}
```

Version 2: parallel for, reduction


```
#include <omp.h>
#include <stdio.h>
/static long num_steps = 100000;
double step;
void main ()
{    int i;       double x, pi, sum = 0.0;
     step = 1.0/(double) num_steps;
    #pragma omp parallel for private(x) reduction(+:sum)
     for (i=1; i<= num_steps; i++){
      x = (i-0.5)*step;
      sum = sum + 4.0/(1.0+x*x);
     }
     pi = sum;
     printf ("pi = %6.8f\n", pi);
}
```

Reduction Options...

- Arithmetic
  - `+ * -`
- Comparison
  - `min max`
- Logical
  - `& && | || ^`
- And now you know why RISC-V has the atomic memory operations:
  - **`amoadd`**, **`amoand`**, **`amoor`**, **`amoxor`**, **`amomax`**, **`amomin`**
    - All but - and * as the reduction can be implemented as single instructions

And in Conclusion, …

- Sequential software is slow software
  - SIMD and MIMD only path to higher performance
- Multithreading increases utilization, Multicore more processors (MIMD)
- OpenMP as simple parallel extension to C
  - Threads, Parallel for, private, critical sections, …
  - ≈ C: small so easy to learn, but not very high level and it's easy to get into trouble